ver.1.1 (2017.9.7)

to robust performance

[mixed sensitivity problem] $\left\| \begin{bmatrix} W_T T \\ W_S S \end{bmatrix} \right\|_\infty < 1$

typical problem
- robust stability
- but nominal performance

robust performance?

[performance optimization (sensitivity minimization)] $\| W_S S \|_\infty < \gamma \to \min$

$\gamma \to \min$

[robust stabilization] $\| W_T T \|_\infty < 1$

property of $\bar{\sigma}$

[H_∞ control problem]

find K s.t.
(i) C.L. is stable
(ii) $\| T_{zw} \|_\infty < 1$

[small gain theorem]

M: stable
$\| M \|_\infty < 1$

[robust stabilization problem]

find K s.t.
(i) C.L. is stable $\forall \Delta$
Δ: stable $\| \Delta \|_\infty \le 1$

less conservative design

[scaled H_∞ control problem]

find W, K s.t.
(i) stability & (ii)

S: sensitivity function
$S := \frac{1}{1+PK}$

T: complementary sensitivity function
$T := \frac{PK}{1+PK}$

$S+T=1$.

y ← P(Δ) ← u ← K ← r

find K s.t.
(i) C.L. is stable $\forall \Delta$
(ii) performance

point to set (class)
$P(s) \to P(\Delta, s)$

[uncertainty model]
① multiplicative
$P(\Delta, s) = P_0(s)(1 + W_t(s)\Delta(s))$
② additive
$P(\Delta, s) = P_0(s) + W(s)\Delta(s)$

$\Delta(s)$: stable
$\| \Delta \|_\infty \le 1$.

post modern control theory 1980~

H_∞ control theory

to learn how to use robust control to design controller for practical system

physical modeling

gap

differential equation

to consider plant uncertainty

Advanced Automation

$\bar{\sigma}$: maximum singular value

$\sigma_i(A) = \sqrt{\lambda_i(A^* A)}$ eigenvalue

$\bar{\sigma}(A) = \max_i \sigma_i(A)$

[H_∞ norm]

$\| G \|_\infty := \begin{cases} \sup_\omega |G(j\omega)| & \cdots \text{SISO} \\ \sup_\omega \bar{\sigma}(G(j\omega)) & \cdots \text{MIMO} \end{cases}$

$\sup_w \frac{\|z\|_2}{\|w\|_2}$ $z = Gw$

classical control theory 1940~

point to set

modern control theory 1960~

objective way (less trial & error)

more mathematical tool

Subjective way (trial & error)

graphical tool for human

to systematical design

for review

- Bode plot
- Nyquist plot

⇒ (·GM (Gain margin), ·PM (Phase ″)) ⇒ robustness for closed-loop stability

Nyquist stability criterion open loop

Gain $|P(j\omega)|$ −20dB/dec. → ω, ω_c

$P(s) = \frac{\omega_c}{s+\omega_c}$

- cost function : $J = \int_0^\infty \{ x^T(t) Q x(t) + u^T(t) R u(t) \} dt$ $Q \ge 0$, $R > 0$
(LQR, LQG, ...)

---- ARE (Algebraic Riccatti Equation)
matrix version
quadratic equation

min

$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = P$ Doyle's notation

$\dot{x} = Ax + Bu$
$y = Cx + Du$

⇒ SSR (State-Space Representation)

TF (Transfer function) ⇒ · SISO (Single Input Single Output) ⇒ · MIMO (Multi Input Multi Output)

- $K(s) = K_p + \frac{K_I}{s} + K_D s$; PID control --- low order ⇒ · high order