

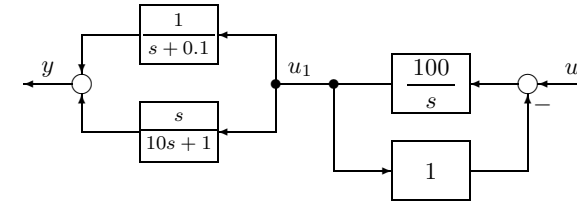
演習 1 解答

問題 1.

問 1. u_1 から y までの伝達関数:

$$\frac{1}{s+0.1} + \frac{s}{10s+1} = \frac{10}{10s+1} + \frac{s}{10s+1} = \frac{s+10}{10s+1}$$

$$C_1(s)C_2(s) = \frac{10}{10s+1} \frac{s+10}{10} = \frac{s+10}{10s+1}$$



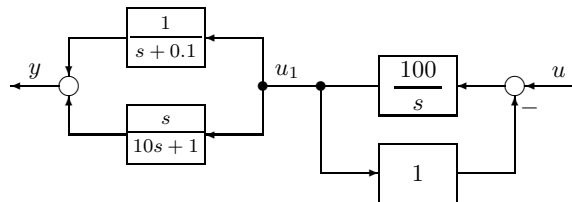
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問題 1.

問 2. u から u_1 までの伝達関数 $C_3(s)$:

$$C_3(s) = \frac{\frac{100}{s}}{1 + \frac{100}{s} \cdot 1} = \frac{100}{s+100}$$



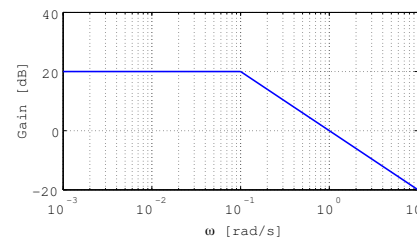
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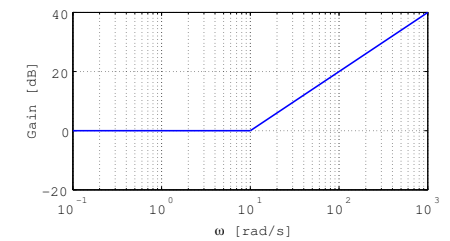
問題 1.

問 3. 周波数伝達関数 $C_1(j\omega)$, $C_2(j\omega)$:

$$C_1(j\omega) = \frac{10}{10j\omega+1}, \quad C_2(j\omega) = 0.1j\omega+1$$



$C_1(j\omega)$



$C_2(j\omega)$

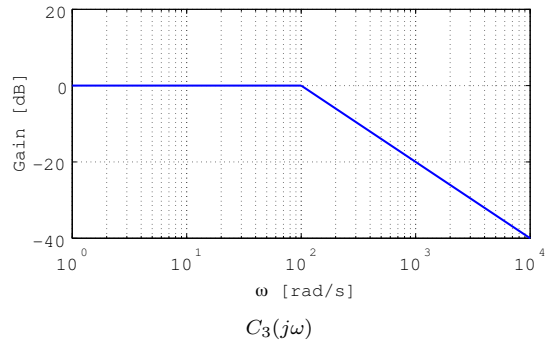
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問題 1.

問 4. 周波数伝達関数 $C_3(j\omega)$:

$$C_3(j\omega) = \frac{100}{j\omega + 100} = \frac{1}{0.01j\omega + 1}$$

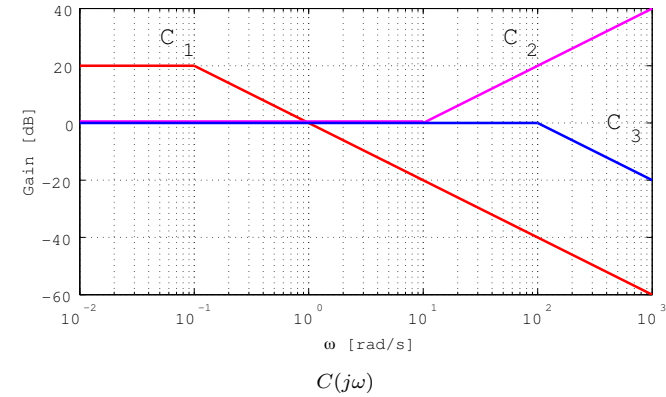


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問題 1.

問 5. $C(j\omega) = C_1(j\omega)C_2(j\omega)C_3(j\omega)$ のゲイン線図:

$$20 \log_{10} |C(j\omega)| = 20 \log_{10} |C_1(j\omega)| + 20 \log_{10} |C_2(j\omega)| + 20 \log_{10} |C_3(j\omega)|$$

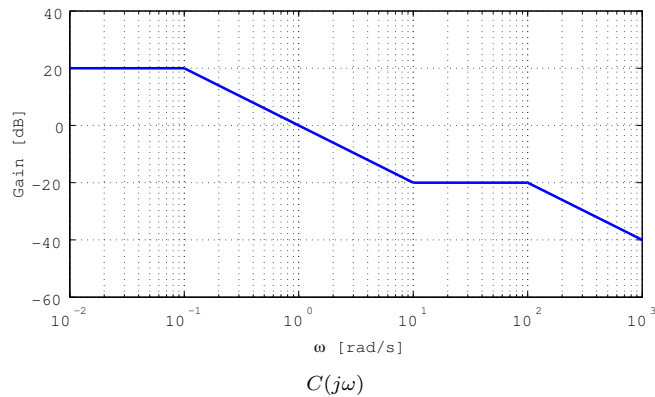


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問題 1.

問 5. $C(j\omega) = C_1(j\omega)C_2(j\omega)C_3(j\omega)$ のゲイン線図:

$$20 \log_{10} |C(j\omega)| = 20 \log_{10} |C_1(j\omega)| + 20 \log_{10} |C_2(j\omega)| + 20 \log_{10} |C_3(j\omega)|$$

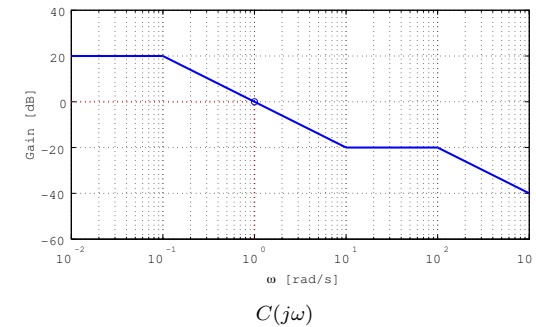


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問題 1.

問 6. $\omega = 1$ のとき $20 \log_{10} |C(j1)| = 0$ [dB]:

$$A_1 = |C(j1)| = 1$$

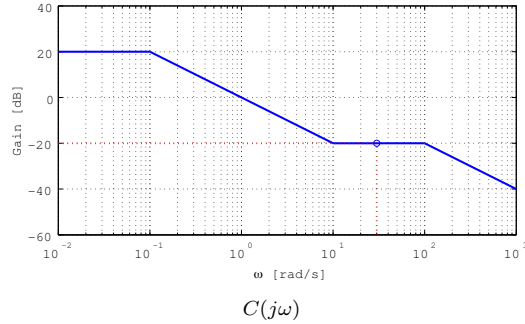


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問題 1.

問 7. $\omega = 30$ のとき $20 \log_{10} |C(j30)| = -20$ [dB]:

$$A_2 = |C(j30)| = \frac{1}{10} = 0.1$$



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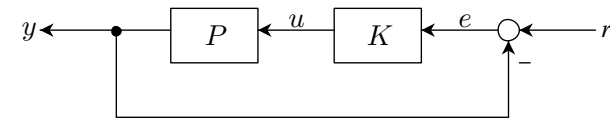
問題 2.

問 1.

$$L(s) = P(s)K(s) = \frac{100}{s(s+20)}$$

$$G_{yr}(s) = \frac{P(s)K(s)}{1 + P(s)K(s)} = \frac{\frac{1}{s(s+20)} 100}{1 + \frac{1}{s(s+20)} 100}$$

$$= \frac{100}{s^2 + 20s + 100} = \frac{100}{(s+10)^2}$$



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問題 2.

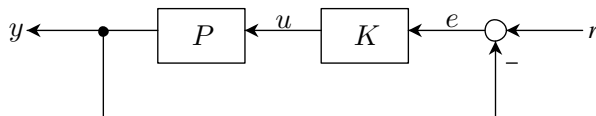
問 1.

$$G_{ur}(s) = \frac{K(s)}{1 + P(s)K(s)} = \frac{100}{1 + \frac{1}{s(s+20)} 100}$$

$$= \frac{100s(s+20)}{s^2 + 20s + 100} = \frac{100s(s+20)}{(s+10)^2}$$

$$G_{er}(s) = \frac{1}{1 + P(s)K(s)} = \frac{1}{1 + \frac{1}{s(s+20)} 100}$$

$$= \frac{s(s+2)}{s^2 + 20s + 100} = \frac{s(s+2)}{(s+10)^2}$$



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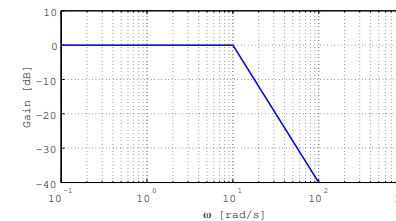
問題 2.

問 2. 周波数伝達関数 $G_{yr}(j\omega)$:

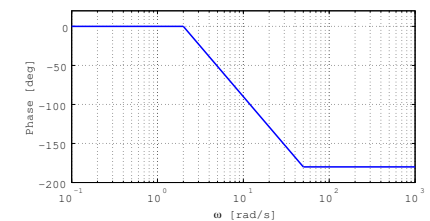
$$G_{yr}(j\omega) = \frac{100}{(j\omega + 10)^2} = \frac{10}{j\omega + 10} \cdot \frac{10}{j\omega + 10} = \frac{1}{0.1j\omega + 1} \cdot \frac{1}{0.1j\omega + 1}$$

位相線図の折点角周波数:

$$\frac{1}{5} \times \frac{1}{T} = \frac{1}{5 \times 0.1} = 2 \text{ [rad/s]} \text{ および } 5 \times \frac{1}{T} = \frac{5}{0.1} = 50 \text{ [rad/s]}$$



$G_{yr}(j\omega)$ のゲイン線図



$G_{yr}(j\omega)$ の位相線図

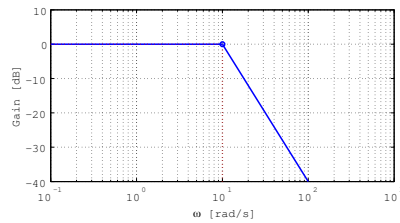
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問題 2.

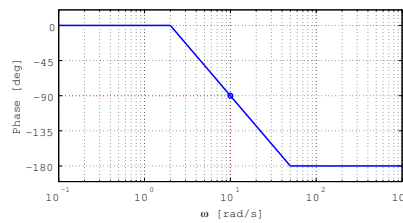
問 3. $\omega = 10$ [rad/s] のとき $20 \log_{10} |G_{yr}(j10)| = 0$ [dB]:

$$A_1 = |G_{yr}(j10)| = 1$$

$$\phi_1 = \angle G_{yr}(j10) = -90 \text{ [deg]}$$



$G_{yr}(j\omega)$ のゲイン線図



$G_{yr}(j\omega)$ の位相線図

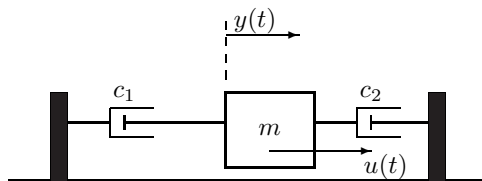
* 計算により求めると $|G_{yr}(j10)| = \frac{1}{2}$



問題 3.

問 1. 運動方程式:

$$m\ddot{y}(t) = u(t) - c_1\dot{y}(t) - c_2\dot{y}(t)$$

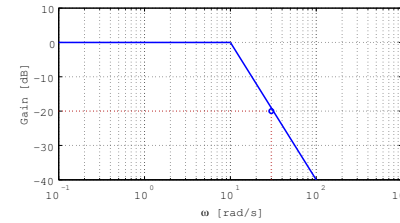


問題 2.

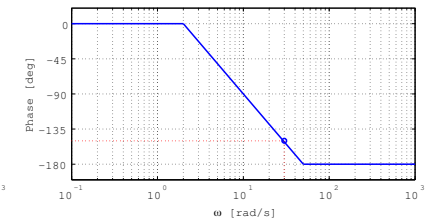
問 3. $\omega = 30$ [rad/s] のとき $20 \log_{10} |C(j30)| = -20$ [dB]:

$$A_1 = |G_{yr}(j30)| = \frac{1}{10} = 0.1$$

$$\phi_1 = \angle G_{yr}(j30) = -150 \text{ [deg]}$$



$G_{yr}(j\omega)$ のゲイン線図



$G_{yr}(j\omega)$ の位相線図



問題 3.

問 2. 運動方程式をラプラス変換:

$$m(s^2y(s) - sy(0) - \dot{y}(0)) = u(s) - c_1(sy(s) - y(0)) - c_2(sy(s) - y(0))$$

初期条件 $y(0) = 0, \dot{y}(0) = 0$:

$$m(s^2y(s) - 0 - 0) = u(s) - c_1(sy(s) - 0) - c_2(sy(s) - 0)$$

$$ms^2y(s) = u(s) - c_1sy(s) - c_2sy(s)$$

伝達関数:

$$ms^2y(s) + c_1sy(s) + c_2sy(s) = u(s)$$

$$\{ms^2 + (c_1 + c_2)s\}y(s) = u(s)$$

$$\frac{y(s)}{u(s)} = \frac{1}{ms^2 + (c_1 + c_2)s} = P(s)$$

$m = 4, c_1 = 1/2, c_2 = 3/2$ を代入:

$$P(s) = \frac{1}{4s^2 + \left(\frac{1}{2} + \frac{3}{2}\right)s} = \frac{1}{4s^2 + 2s}$$



問題 3.

問 3. 振幅 A_1 は $\omega = 1/2$ のときのゲイン $|P(j\omega)|$:

$$|P(j\omega)| = \frac{|1|}{|-4\omega^2 + 2j\omega|} = \frac{1}{\sqrt{(-4\omega^2)^2 + (2\omega)^2}}$$

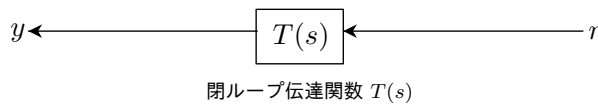
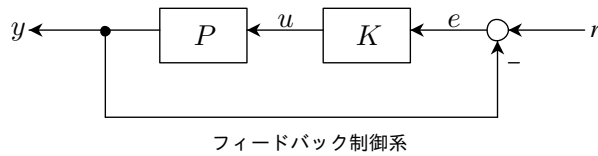
$\omega = 1/2$ を代入:

$$A_1 = \left| P\left(\frac{1}{2}j\right) \right| = \frac{1}{\sqrt{\left(-4\left(\frac{1}{2}\right)^2\right)^2 + \left(2\left(\frac{1}{2}\right)\right)^2}} = \frac{1}{\sqrt{2}}$$

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問題 3.

問 4.

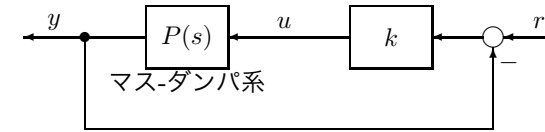


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問題 3.

問 4.

$$T(s) = \frac{P(s)k}{1 + P(s)k} = \frac{\frac{1}{4s^2 + 2s}k}{1 + \frac{1}{4s^2 + 2s}k} = \frac{k}{4s^2 + 2s + k}$$



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問題 3.

問 5. $A_2 = 10$ は $\omega = \sqrt{\frac{k}{m}} = \frac{\sqrt{k}}{2}$ のときのゲイン $|T(j\omega)|$:

$$|T(j\omega)| = \frac{|k|}{|-4\omega^2 + 2j\omega + k|} = \frac{k}{\sqrt{(-4\omega^2 + k)^2 + (2\omega)^2}}$$

$\omega = \frac{\sqrt{k}}{2}$ を代入:

$$A_2 = \left| T\left(\frac{\sqrt{k}}{2}j\right) \right| = \frac{k}{\sqrt{\left(-4\left(\frac{\sqrt{k}}{2}\right)^2 + k\right)^2 + \left(2\left(\frac{\sqrt{k}}{2}\right)\right)^2}} = \frac{k}{\sqrt{k}} = \sqrt{k}$$

($A_2 =$) $10 = \sqrt{k}$ であるから、 $k = 100$ とすればよい。

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