$$
\begin{aligned}
J &= \int_0^\infty \left\{x^T Q x + u^T R u\right\} dt && (1)\\
&= \int_0^\infty \left[x^T \left\{-A^T P - PA + PBR^{-1}B^T P\right\} x + u^T R u - u^T B^T P x - x^T P B u + u^T B^T P x + x^T P B u\right] dt && (2)\\
&= \int_0^\infty \left[-\left\{x^T A^T + u^T B^T\right\} P x - x^T P \left\{Ax + Bu\right\} + \left\{x^T PBR^{-1} + u^T\right\} R \left\{R^{-1}B^T P x + u\right\}\right] dt && (3)\\
&= -\int_0^\infty \dot{x}^T P x dt - \int_0^\infty x^T P \dot{x} dt + \int_0^\infty \left\{x^T PBR^{-1} + u^T\right\} R \left\{R^{-1}B^T P x + u\right\} dt && (4)\\
&= -\left[x^T P x\right]_0^\infty + \int_0^\infty x^T P \dot{x} dt - \int_0^\infty x^T P \dot{x} dt + \int_0^\infty \left\{x^T PBR^{-1} + u^T\right\} R \left\{R^{-1}B^T P x + u\right\} dt && (5)\\
&= x(0)^T P x(0) + \int_0^\infty \left\{x^T PBR^{-1} + u^T\right\} R \left\{R^{-1}B^T P x + u\right\} dt && (6)
\end{aligned}
$$