Speed control of two inertia system with servo motor

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November 28, 2019

1 Experimental setup

Fig. 1 and Table 1 show the block diagram and the equipments of the experimental system. The apparatus is mainly composed of two servo motors and a inertia-load disc connected with two couplings, a low-stiffness coupling (long shaft) and a rigid coupling. The servo motors have rotary encoders whose output pulse signal is processed by a counter to generate rotational position. Rotational speed is approximately measured by evaluating difference of rotational pulses. The driving torque of the servo motor is specified by analog voltage generated by D/A converter. Controllers are implemented as a real-time task of RT-Linux on PC.

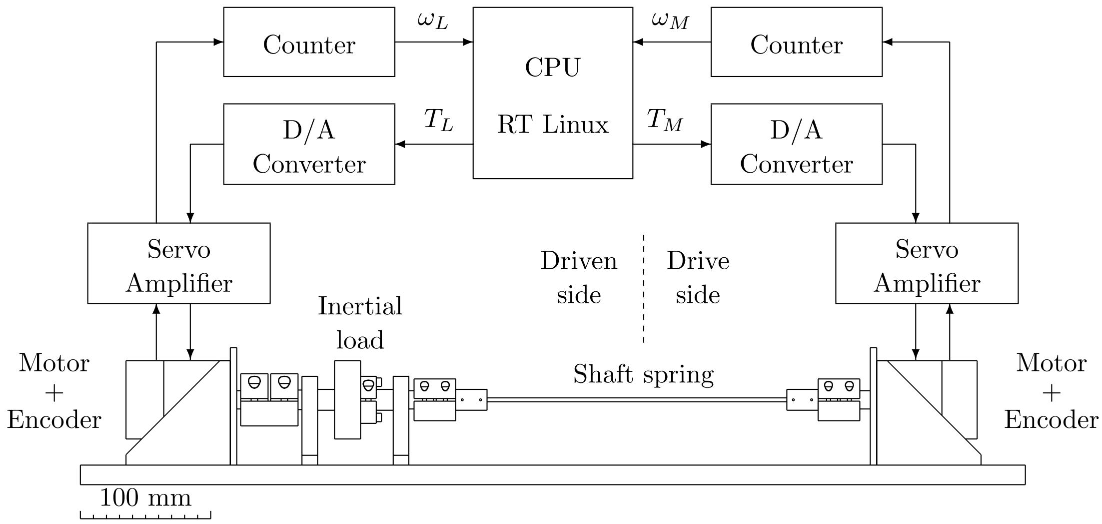

Figure 1: Block diagram of experimental apparatus

Table 1: Experimental equipments

PC	Dell Dimension 2100 / Fedora Core 1 (RTLinux 3.2-pre3, Linux kernel 2.4.22)
D/A	CONTEC DA12-4(PCI) (12bit, 10μs)
counter	CONTEC CNT24-4(PCI)H (24bit, 1MHz)
PIO	CONTEC PIO-32/32T(PCI) (Parallel input output, 32bit 200ns)
A/D	CONTEC AD12-16(PCI) (12bit, $\pm$5V 10μs)
Driving & driven motors	YASKAWA ELECTRIC CORPORATION SGD7S-1R6A00A, SGM7J-02AFA21 (rated power: 200 W, rated torque: 0.637 Nm(max 2.23 Nm), rotor inertia moment: 0.263×10^{-4} kg $\cdot$ m^2), speed/position detector: 20-bit encoder
Shaft spring	ϕ 3 mm $\times$ L 230 mm (The shaft is 250 mm long.) material: SUS304
Dynamic torque meter	UTMⅡ -20Nm (Max torque range: $\pm$20N $\cdot$ m, Max output voltage:$\pm$5vDC))
Inertial load	ϕ 60 mm $\times$ T 13 mm, ϕ 80 mm $\times$ T 20 mm material: SS400

2 Derivation of physical model

For the given practical system, let us consider to derive a state-space representation from T_M to ω_M. The system can be regarded as a simple two-inertia system depicted in Fig. 2.

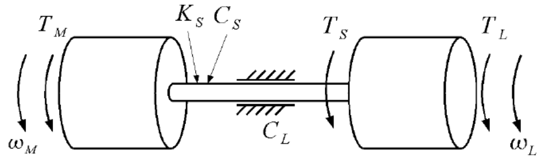

Figure 2: Two Inertia System

Define the following variables:
θ_M : Rotational angle of driving motor (rad)
ω_M : Rotational speed of driving motor (rad/s)
J_M : Moment of inertia of driving motor (Kg m^2)
T_M : Driving torque (Nm)
θ_L : Rotational angle of load (rad)
ω_L : Rotational speed of load (rad/s)
J_L : Moment of inertia of load (Kg m^2)
T_L : Disturbance torque of load (Nm)
C_L : Damping coefficient due to friction on load
K_S : Torsional stiffness of shaft (Nm/rad)
C_S : Torsional damping coefficient of shaft
T_S : Torsional torque on shaft (Nm)

Let θ_r be a relative angle of the motor and the load as $\theta_r := \theta_M - \theta_L$, then equation of motion of the system is given as following:

$$\begin{aligned} J_M\dot{\omega}_M &= T_M - T_S && (1)\\ T_S &= K_S\theta_r + C_S(\omega_M - \omega_L) && (2)\\ J_L\dot{\omega}_L &= T_L + T_S - C_L\omega_L && (3) \end{aligned}$$

Let state-space variable x be

$$x := \begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix}. \quad (4)$$

State-space representation of the system from $\begin{bmatrix} T_L & T_M \end{bmatrix}^T$ to $\begin{bmatrix} \omega_L & \omega_M \end{bmatrix}^T$ is given by

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} &= \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K_S}{J_M} & -\frac{C_S}{J_M} & \frac{C_S}{J_M} \\ \frac{K_S}{J_L} & \frac{C_S}{J_L} & -\frac{C_S+C_L}{J_L} \end{bmatrix}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{J_M} \\ \frac{1}{J_L} & 0 \end{bmatrix}\begin{bmatrix} T_L \\ T_M \end{bmatrix} && (5)\\ \begin{bmatrix} \omega_L \\ \omega_M \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} && (6) \end{aligned}$$

Torsional resonance frequency f_r and anti-resonance frequency f_a are given respectively as follows:

$$f_r = \frac{1}{2\pi}\sqrt{K_S\left(\frac{1}{J_L} + \frac{1}{J_M}\right)}, \quad f_a = \frac{1}{2\pi}\sqrt{\frac{K_S}{J_L}} \quad (7)$$