[初期値が非零の場合は？].

$$x''-5x'+6x=e^t$$

方法1：ラプラス変換を使わない場合．

$$x=x_1+x_2$$

$$x_1''-5x_1'+6x_1=0.$$

$$\lambda^2-5\lambda+6=0.$$

$$(\lambda-2)(\lambda-3)=0.,\quad \lambda=2,3.$$

$$x_1(t)=C_1e^{2t}+C_2e^{3t}\quad C_1, C_2 \text{ 定数}$$

$x_2=Ae^t$ とおくと

$$Ae^t-5Ae^t+6Ae^t=e^t$$

$$(1-5+6)A=1,\quad A=\frac{1}{2}.$$

$$\therefore x_2=\frac{1}{2}e^t.$$

$$\therefore x=C_1e^{2t}+C_2e^{3t}+\frac{1}{2}e^t$$

変更なし

$$x(0)=C_1+C_2+\frac{1}{2}\quad \text{——①}$$

$$x'(0)=2C_1+3C_2+\frac{1}{2}$$

$$-)\quad 2x(0)=2C_1+2C_2+1\quad \text{——①×2}$$

$$x'(0)-2x(0)=C_2-\frac{1}{2}$$

$$\therefore C_2=\frac{1}{2}+x'(0)-2x(0)$$

①より

$$C_1=x(0)-C_2-\frac{1}{2}$$

$$=x(0)-\frac{1}{2}-x'(0)+2x(0)-\frac{1}{2}$$

$$=3x(0)-x'(0)-1.$$

$$\therefore x(t)=\left(3x(0)-x'(0)-1\right)e^{2t}+\left(\frac{1}{2}+x'(0)-2x(0)\right)e^{3t}+\frac{1}{2}e^t \quad //$$

方法2：ラプラス変換を使う場合．

$$s^2X(s)-sx(0)-x'(0)-5\left(sX(s)-x(0)\right)+6X(s)=\frac{1}{s-1}.$$

$$(s^2-5s+6)X(s)=\frac{1}{s-1}+sx(0)+x'(0)-5x(0)$$

$$(s-2)(s-3)X(s)=\frac{1+(s-1)\left(sx(0)+x'(0)-5x(0)\right)}{s-1}$$

$$X(s)=\frac{1+(s-1)\left(sx(0)+x'(0)-5x(0)\right)}{(s-1)(s-2)(s-3)}$$

$$=\frac{A}{s-1}+\frac{B}{s-2}+\frac{C}{s-3}\quad \text{とおく．}$$

$$A=\left.\frac{1+(s-1)\left(sx(0)+x'(0)-5x(0)\right)}{(s-2)(s-3)}\right|_{s=1}$$

$$=\frac{1}{-1\cdot(-2)}=\frac{1}{2}.$$

$$B=\left.\frac{1+(s-1)\left(sx(0)+x'(0)-5x(0)\right)}{(s-1)(s-3)}\right|_{s=2}$$

$$=\frac{1+2x(0)-5x(0)+x'(0)}{1\cdot(-1)}$$

$$=3x(0)-x'(0)-1.$$

$$C=\left.\frac{1+(s-1)\left(sx(0)+x'(0)-5x(0)\right)}{(s-1)(s-2)}\right|_{s=3}$$

$$=\frac{1+2\left(3x(0)+x'(0)-5x(0)\right)}{2\cdot 1}$$

$$=\frac{1}{2}-2x(0)+x'(0)$$

$$x(t)=\frac{1}{2}e^t+\left(3x(0)-x'(0)-1\right)e^{2t}+\left(\frac{1}{2}-2x(0)+x'(0)\right)e^{3t} \quad //$$