Speed control of two inertia system with servo motor

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1 Experimental setup

Figure1 and Table1 show a block diagram and instruments of experimental apparatus.

The control target is mainly composed of two servomotors, rotating shafts, and an inertia-load disk, and couplings. The right-hand-side servo motor is for driving, which is connected to the left-hand-side driven load by a coupling whose spring stiffness is $K_S = 523$Nm/rad. The driven load is composed of an inertia disk and a servo motor which are connected by a rigid coupling. The left-hand-side servo motor is used to simulate disturbance torque occurred in driven side. The total inertia moment of driving side and driven side are $J_M = 0.17 \times 10^{-4}$kgm^2 and $J_L = 2.04 \times 10^{-4}$kgm^2, respectively.

Each servo motor has a rotary encoder whose output pulse signal is processed by a counter to generate rotational position. Rotational speed is approximately measured by evaluating difference of rotational pulses. The driving torque of each servo motor is specified by analog voltage generated by D/A converter. Controllers are implemented as a real-time task of RT-Linux on PC.

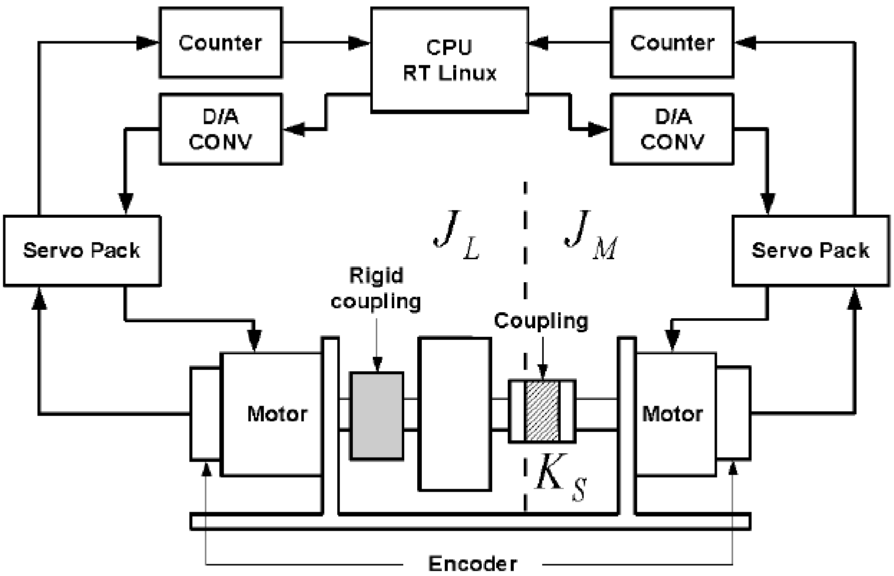

Figure 1: Experimental apparatus

Table 1: Experimental equipments

Disturbance motor	ORIENTAL MOTOR 0.116×10^{-4}kgm^2, 200W, 1.9Nm(max), 10000ppr
Driving motor	YASUKAWA ELEC. SGMAV-02A, SGDV-1R6A 0.088×10^{-4}kgm^2, 200W, 1.9Nm(max), 65536ppr
PC	DELL Dimension 2100 (Celeron 1000MHz) RT-Linux 3.1 / Fedora core 1 (kernel 2.4.18)
D/A	Interface PCI-360116 (2ch, 16bit, 10μsec)
Counter	CONTEC CNT24-4(PCI)H (4ch, 24bit, 1MHz)

2 Derivation of plant model

Consider a two inertia system dipicted in Fig. 2. 2)

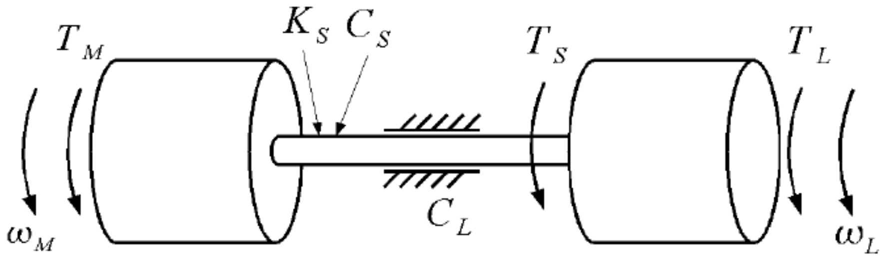

Figure 2: Two Inertia System

Denote variables as following:
θ_M : Rotational angle of driving motor [rad]
ω_M : Rotational speed of driving motor [rad/sec]
J_M : Moment of inertia of driving motor [Kg m^2]
T_M : Driving torque [Nm]
θ_L : Rotational angle of load [rad]
ω_L : Rotational speed of load [rad/sec]
J_L : Moment of inertia of load [Kg m^2]
T_L : Disturbance torque of load [Nm]
C_L : Damping coefficient due to friction on load
K_S : Torsional stiffness of shaft [Nm / rad]
C_S : Torsional damping coefficient of shaft
T_S : Torsional torque on shaft [NM]

Let θ_r be a relative angle of the motor and the load as $\theta_r := \theta_M - \theta_L$, then equation of motion of the system is given as following:

$$J_M\dot{\omega}_M = T_M - T_S \quad (1)$$
$$T_S = K_S\theta_r + C_S(\omega_M - \omega_L) \quad (2)$$
$$J_L\dot{\omega}_L = T_L + T_S - C_L\omega_L \quad (3)$$

Let state-space variable x be

$$x := \begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix}. \quad (4)$$

State-space representation of the system from T_M to ω_M is given by

$$\frac{d}{dt}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -\frac{K_S}{J_M} & -\frac{C_S}{J_M} & \frac{C_S}{J_M} \\ \frac{K_S}{J_L} & \frac{C_S}{J_L} & -\frac{C_S+C_L}{J_L} \end{bmatrix}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_M} \\ 0 \end{bmatrix} T_M \quad (5)$$

$$\omega_M = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \theta_r \\ \omega_M \\ \omega_L \end{bmatrix} \quad (6)$$

Torsional resonance frequency f_r and anti-resonance frequency f_a are given respectively as follows:

$$f_r = \frac{1}{2\pi}\sqrt{K_S\left(\frac{1}{J_L} + \frac{1}{J_M}\right)}, \quad f_a = \frac{1}{2\pi}\sqrt{\frac{K_S}{J_L}} \quad (7)$$