Math. Preliminaries

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$$A \in \mathbf{R}^{n \times n} \tag{1}$$

$$A_c \in \mathbf{C}^{n \times n} \tag{2}$$

1 eigen value

$$Ax_i = \lambda_i x_i, \quad i = 1, \ldots, n \tag{3}$$

λ_i: eigen value, x_i: eigen vector

Diagonalization

$$A = X \Lambda X^{-1} \tag{4}$$

Q. What are eigen values and vectors of the following matrix A.

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \tag{5}$$

2 Unitary Matrix

"Rotation" matrix

A real matrix A is unitary if $A^T = A^{-1}$

A complex matrix A_c is unitary if $\bar{A}^T = A^{-1}$

3 Singular Value

$$\sigma_i = \sqrt{\lambda_i(A^T A)}, \quad i = 1, \ldots, n \tag{6}$$

Singular Value Decomposition (SVD)

$$A = USV^T \tag{7}$$

$$U, V : \text{unitary matrix} \tag{8}$$

$$S : \text{diag}(\sigma_1, \ldots, \sigma_n) \tag{9}$$