Math. Preliminaries

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1 nortations

$$N := \text{natural numbers} \quad (1)$$
$$Z := \text{integer} \quad (2)$$
$$R := \text{real numbers} \quad (3)$$
$$C := \text{complex numbers} \quad (4)$$
$$(K := R \text{ or } C) \quad (5)$$

2 vector space

2.1 definiton

X is a vector space (linear space) if

$\forall x, y, z \in X$, $\forall a, b \in K$

1. $x + y \in X$
2. $(x + y) + z = x + (y + z)$
3. $x + y = y + x$
4. $\exists^1 0 \in X$, s.t. $0 + x = x$
5. $\exists^1 -x \in X$, s.t. $(-x) + x = 0$
6. $(a + b)x = ax + bx$
7. $a(x + y) = ax + ay$
8. $(ab)x = a(bx)$
9. $1x = x$

2.2 note

X is called a real vector space if $K = R$.
X is called a complexl vector space if $K = C$.

3 norm in scaler vector space

3.1 definiton

The function $\|\cdot\|(x \in X \to \|x\|)$ is *norm* if $\|\cdot\| : X \to [0, \infty)$ and

$$\|x\| = 0 \;\Leftrightarrow\; x = 0 \quad (6)$$
$$\|\alpha x\| = |\alpha|\|x\|,\ \forall \alpha \in K \quad (7)$$
$$\|x + y\| \leq \|x\| + \|y\| \quad (8)$$

3.2 exercise

$$\|-1.1\| = |-1.1| =? \quad (x \in R) \quad (9)$$
$$\|3 + 4j\| = ? \quad (x \in C) \quad (10)$$

extension

$$\|\sin t\| = ? \quad (11)$$
$$\left\| \frac{1}{s^2 + 2s + 3} \right\| = ? \quad (12)$$

3.3 L_p-norm

$\int$: Lebesgue integral
$x(t) : t \in R \mapsto R$ (continuous-time signal)

$$\|x(t)\|_p := \left(\int_{-\infty}^{\infty} |x(\tau)|^p d\tau \right)^{1/p},\ 1 \leq p < \infty \quad (13)$$
$$\|x(t)\|_2 := \left(\int_{-\infty}^{\infty} |x(\tau)|^2 d\tau \right)^{1/2} \quad (14)$$
$$\|x(t)\|_\infty := \sup_{t \in [-\infty, \infty]} |x(t)| \quad (15)$$
$$\lim_{p \to \infty} \|x(t)\|_p \to \|x(t)\|_\infty \quad (16)$$

$L_p(a, b)$-norm

$$\|x(t)\|_{p(a,b)} := \left(\int_a^b |x(\tau)|^p d\tau \right)^{1/p} \quad (17)$$

3.4 exercise

$$
\begin{aligned}
\|\sin t\|_1 &= ? && (18)\\
\|\sin t\|_2 &= ? && (19)\\
\|\sin t\|_\infty &= ? && (20)\\
\|\sin t\|_{1(0,2\pi)} &= ? && (21)\\
\|\sin t\|_{2(0,2\pi)} &= ? && (22)\\
\|\sin t\|_{\infty(0,2\pi)} &= ? && (23)
\end{aligned}
$$

3.5 induced norm

$G : X \to X$: a L_2 stable system,

$$
\|G\|_\infty := \sup_{u(t)\in L_2} \frac{\|Gu(t)\|_2}{\|u(t)\|_2} \tag{24}
$$

This means that the H_∞ norm is a measure of the worst case response. If G is linear

$$
\|G\|_\infty := \sup_{\|u(t)\|=1} \|Gu(t)\|_2 \tag{25}
$$

References

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[3] K.Zhou, J.C.Doyle, and K.Glover, “Robust and Optimal Control”, Prentice Hall, (1996)